REAL-TIME RIGID BODY SIMULATIONS OF SOME 'CLASSICAL MECHANICS TOYS'

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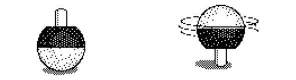
ABSTRACT

We present the first rigid body real-time simulations of the dynamics of 1. the classical tippe-top, 2. the Celtic wobblestone, 3. a spinning ellipsoid and 4. a spinning coin with a hole. The simulations are carried out with two different approaches representing the state-of-the-art in rigid body simulation and both are especially suitable for certain unilateral contact situations. On the one hand we use an 'impulse-based' and on the other hand a 'constraint-based' simulation technique. The behaviour of the toys is very much dependent on dynamic, static and rolling friction effects. That is why they are very appropriate evaluation examples for the friction and also collision modeling capabilities of the two approaches. We present the simulation results, make a comparison and draw conclusions concerning their physical correctness and applicability in our module for the simulation of rigid body dynamics in virtual reality environments. The industrial application background of the simulation techniques is also sketched.

1 INTRODUCTION



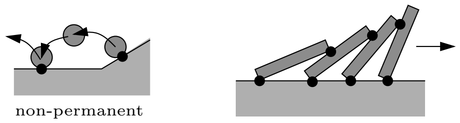
At all times toys like the tippe-top above fascinated children as well as scientists. In the picture below you can see N. Bohr and W. Pauli, the two famous Nobel-prize winning physicists, watching the strange behaviour of a tippe-top at a time its behaviour was not understood at all.

Cohen described [Co77] the scientist's fascination for this toy with the words, *'The tippe-top's motion constitutes the sort of phenomenon abundant in physics,*

for which a simple physical analysis reveals the underlying principles. Yet for which a detailed and rigorous solution (which may require the use of computing machines) is necessary to confirm the analysis.'. Our interest in these toys was initially motivated by the need for some experimental evaluation examples for the friction modeling capabilities of the developmental real-time rigid body simulation tool GALILEO . It is part of the virtual reality software platform DBView developed by the Virtual Reality Competence Center (VRCC) of Daimler-Benz Research. The physical modeling of virtual objects and the simulation of rigid body dynamics in virtual environments will play a very important role throughout the whole manufacturing- and engineering-process. No matter whether automotive, aircraft or railway industry, simulation in the early product development cycles becomes more and more important, that already is our experience. Virtual Reality helps to shorten product development times and therefore cuts costs. The simulation of mechanical systems with unilateral contacts is very useful in the industrial field e.g. when it comes to the simulation of fitting-operations in engine-design. Beyond that the physically correct behaviour of objects in virtual worlds helps to increase the feeling of immersion for the user.

The focus of the system is on the simulation of unilateral contact situations. We present the first rigid body simulations for the above, so to speak, 'classical mechanics toys'. Up to now all simulations of these objects were carried out for specific geometric and dynamic configurations of the toys with partly questionable friction models. As the equations of motion were always adapted to the special configurations of the objects and the plane they were moving on, these simulations were no general approaches to rigid body systems. In our virtual reality tool we have physically modeled virtual objects which move in the virtual environment according to the Newton-Euler equations. A collision detection module passes the new contact configurations in every frame to one of the two simulation approaches. Using the contact information the collisions are physically modeled and the resulting new motions after the collisions are determined and visualized.

Generally unilateral contact situations can be classified according to the picture below.



The distinction of the simulation techniques in two categories is not only justified by the observation of the occurring real contact situations, but also by the fact that for single contact point configurations a 'physically much more correct modeling' is possible as it is for multibody problems (see classical problems with e.g. energy-gains, transition from static to dynamic friction, paradoxa of Painlevé, static indeterminacy, jamming and wedging). The impulse-based technique as described by Mirtich and Canny ([MC94], [MC95]), based on Stronge [St90], Keller [Ke86] and Hahn [Ha88], is especially suitable for the simulation of temporary contacts at a single contact point (see example of the hopping ball). The constraint-based technique according to Sauer and Schömer [SS98b] is based on a work of Stewart and Trinkle ([ST95a],[ST95b]), who themselves relied on [Mo86] and [MM93]. It rather is a simulation approach for problems with multiple permanent contacts between multiple objects (see example of dominoes).

One of the open problems in the field of rigid body simulations is the design and implementation of a hybrid simulation system that combines both types of simulation paradigms. The solution of this problem is the long-term scientific objective of the GALILEO-simulation environment.

2 THE TWO SIMULATION TECHNIQUES

In the following subsections we briefly sketch the two simulation techniques used in the simulation system. Because we cannot go into the mathematical and algorithmic details here, we recommend the interested reader to read the literature ([SS98a], [SS98b]).

2.1 The impulse-based approach

The impulse-based method always determines the behaviour of two objects in contact at a single contact point ('local interaction method'). If two objects collide at multiple contact points a temporal sequence of single contact point collisions is simulated instead. To get the impulses at the contact point, variations (dependent on the acting friction forces and the phase of the collision process) of the following differential equation must be integrated

$$\frac{d}{d\gamma}\boldsymbol{u}(\gamma) = \mathbf{K}\frac{d}{d\gamma}\boldsymbol{p}(\gamma)\ . \tag{1}$$

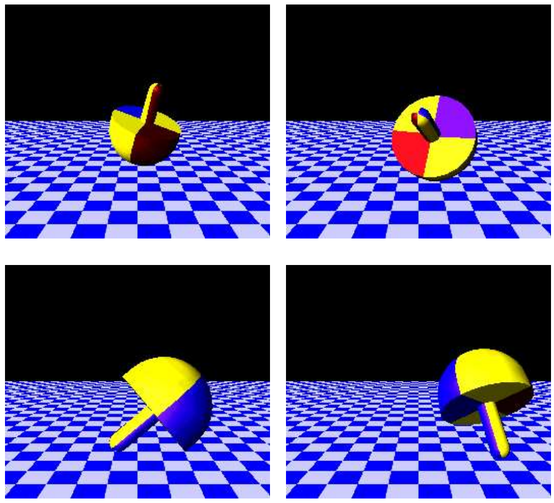


Figure 1: Some snapshots from the tippe-top simulation with the 'LCP-method' of the GALILEO-module of DBView. In the picture below left you see the two contact point situation.

Thereby $\boldsymbol{u}$ is the relative velocity of the contact point, $\boldsymbol{p}$ is the impulse at the contact point exerted on one of the two object ($-\boldsymbol{p}$ acts on the other one), $\mathbf{K} = \left(\frac{1}{m_1}+\frac{1}{m_2}\right)\mathbf{E} - \left(\mathbf{r}_1^\times \mathbf{I}_1^{-1}\mathbf{r}_1^\times + \mathbf{r}_2^\times \mathbf{I}_2^{-1}\mathbf{r}_2^\times\right)$ is the collision matrix build up from the object masses m_i, the inertia matrices $\mathbf{I}_i$ and the vectors $\boldsymbol{r}_i$ from the centers of mass to the contact point ($i = 1, 2$). $\gamma \in \mathbb{R}$ is the integration parameter and $\mathbf{E}$ the identity matrix. Using the solution of (1) the new linear and angular velocities of the colliding objects and following from that their new positions and orientations are determined. Sliding and sticking friction are modeled according to Coulomb. The collision process itself consists of a compression phase followed by a restitution phase according to the energetic internal dissipation hypothesis of Stronge [St90]. The coefficient of restitution defined in this way is just a simple mathematical modeling of elasticity, the virtual objects themselves are rigid and their collision has only infinitesimal small temporal extension. Between two collisions objects always perform ballistic motions. The approach tracks the relative contact velocity, which is not constant during the compression and restitution phase. Because of this tracking and the additional use of the Stronge energetic hypothesis (instead of e.g. the classical Poisson hypothesis) the approach has not the well-known problem of negative energy dissipation that most of the other contact force techniques have. Another classical problem is to find the correct transitions between sticking and sliding friction and vice versa. The impulse-based method correctly simulates these transitions. For some special cases the integration of (1) can be replaced by analytical expressions for the contact velocity at the end of the collision. This approach is, concerning the simulation of single contact problems, the physically most correct one. But when it comes to the simulation of multibody systems the technique of treating simultaneous contacts as a sequence of contacts is not very efficient. That is why in this case we make use of the following simulation approach.

2.2 The constraint-based approach

In contrast to the impulse-based method the constraint-based approach, as described in [SS98b], is designed to determine the dynamical behaviour of a system of rigid bodies with $K \in \mathbb{N}$ unilateral contacts in the presence of friction. The rigid objects are modeled as polyhedral objects or as objects with curved surfaces. In order to perform a time step during the integration of the equations of motion, we calculate the contact forces/impulses by means of a nonlinear complementarity problem (NCP). Substituting the Coulomb friction cones by faceted friction pyramids and linearizing the distance functions for each contact, we are able to transform the original NCP into a linear complementarity problem (LCP) as proposed by [ST95b]. Using a fixpoint-iteration scheme the contact forces/impulses resulting from the LCP formulation quickly converge to the desired solution of the original NCP. The solution of the LCP is determined by the classical combinatorial Lemke-algorithm (see [CPS92]). The linear complementarity problem (LCP) has the form

$$\mathbf{A}\boldsymbol{x} + \boldsymbol{b} \geq \mathbf{0}, \quad \boldsymbol{x} \geq 0, \quad (\mathbf{A}\boldsymbol{x} + \boldsymbol{b})^T \boldsymbol{x} \geq 0 \qquad (2)$$

with $\mathbf{A} \in \mathbb{R}^{(\eta+2)\mathrm{K} \times (\eta+2)\mathrm{K}}$. The Coulomb friction cone is linearized by a friction pyramid with $\eta \in \mathbb{N}$ many facets. As you can see the number of facets of the friction pyramid and the number of contact points determine the dimension of the matrix $\mathbf{A}$. The solution of the LCP determines the contact force and torque exerted on every object. Inserting these forces into the discretized Newton-Euler equations then delivers the new positions and orientations of the objects.

Notice that the LCP approach is not the well-known standard LCP formulation with all its drawbacks based on a complementarity relation between the conditions imposed on the normal accelerations of the contact points and the resulting contact forces as described e.g. by Baraff [Ba94] and others. We have a complicated complementarity relation between two types of initially nonlinear constraints that have to be enforced for every contact point: 1. the Coulomb friction constraint and 2. the geometrical contact condition. The simulations are energy consistent and do not have the classical problems described by Painlevé or the numerical problems of the traditional LCP formulations. The existence of a solution is guaranteed and although the set of resulting contact forces is not always unique, the physical behaviour is.

The impulse-based and the constraint-based method are generally capable of simulating the dynamics of bilateral constraints too. Even if the examples following in the next sections are quite simple, concerning their geometrical complexity (number of objects, number of faces involved), the approaches themselves are suitable for the simulation of complex mechanical systems. But this is not the focus of the paper. We are more interested in qualitative than quantitative aspects of the simulation techniques here.

3 THE SIMULATION OF THE SPHERE WITH AN ECCENTRIC CENTER OF MASS AND OF THE TIPPE-TOP

The GALILEO-system was designed for the solution of multibody simulation tasks, but already the simulation of the mechanical behaviour of just one object can be quite complicated. A very famous example is the tippe-top. It appears to be stable when spun slowly, but if it is spun faster it overturns and spins on its stick. Roughly speaking, the friction at the contact point(s) between the tippe-top and the plane is the reason for the inversion. We cannot go into the physical details of the tippe-top here and recommend the literature. Classical papers are [Br52], [Hu52], [Sy52], [Pl54], [De55]. [Co77] and [Or94] carried out computer-simulations. The most important former work has been presented by Kane and Levinson [KL78]. We reimplemented their approach to check our own results. But up to now all the simulations done just imitate the behaviour of a sphere with an eccentric center of mass by integrating the equations of motion that are adapted to the special geometry of the sphere. Beyond that the results of the simulations are partly incorrect because of wrong friction modeling (see e.g. the comments of [KL78] on [Co77]).

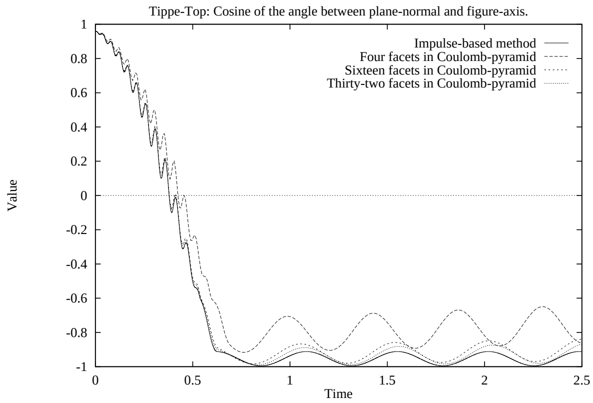


Figure 2: The cosine of the angle between the plane-normal and the figure-axis of the top shows the overturning and up to time $t \approx 0.6$ the nutational motion of the top. The uppermost curve corresponds to the simulation with the constraint-based method with four facets in the friction-pyramid, the two curves below to the same approach with sixteen and thirty-two facets in the Coulomb-pyramid. The curve at the bottom corresponds to the impulse-based simulation.

As you can see in figure 1 our system is able to simulate the tipping behaviour with both approaches. Beyond that the rigid body simulations satisfy the real-time demands. For example in the simulation of the sphere with the eccentric center of mass the following computing-times for the solution of the LCP were achieved on a SGI Infinite Reality (one R10000 processor, averaging of 10000 tests):

dimension of **A**	6	10	18
time in milliseconds	0.765	1.202	2.764
dimension of **A**	34	66	130
time in milliseconds	7.941	27.464	110.923

For the impulse-based method the real-time demands are easily met, too. The results we got for the simulation of the sphere with eccentric center of mass are identical to those of Kane and Levinson. The LCP approach simulates the effect with an arbitrary number of facets in the Coulomb friction pyramid as you can see in figure 2. Figure 3 shows the forces determined by the LCP approach and the change of the contact point (change of index from 1 to 2 in the figure) at time $t \approx 0.6$. Figure 4 shows the three-dimensional motion of the center of mass. Figure 5 compares the normal coordinate of the center of mass in both simulation approaches during a simulation with the same initial configuration. Using the impulse-based technique every collision process is simulated with the best available model, but between two collisions the objects always exhibit a ballistic motion. In the constraint-based approach the complementarity conditions prohibit such a behaviour. One should not forget that the constraint-based method is essentially designed for and certainly has its strength in the simulation of 'multiple body multiple contact point problems'. But the impressing thing about the constraint-based simulation is, that in spite of this fact, the curves tend towards the simulation results of the impulse-based method if the number of facets in the friction pyramid is increased.

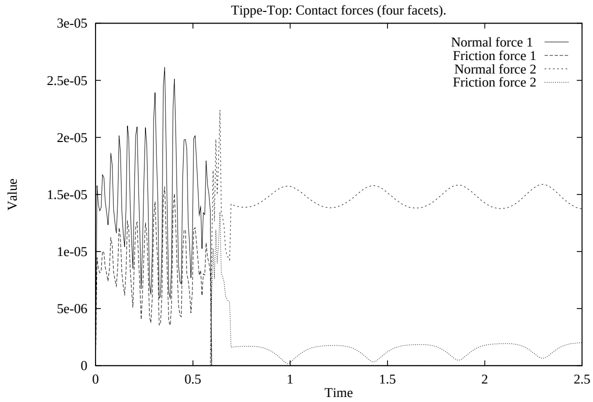


Figure 3: The two uppermost curves show the normal forces exerted on the top at contact point 1 and later on at contact point 2. The two curves below correspond to the friction forces.

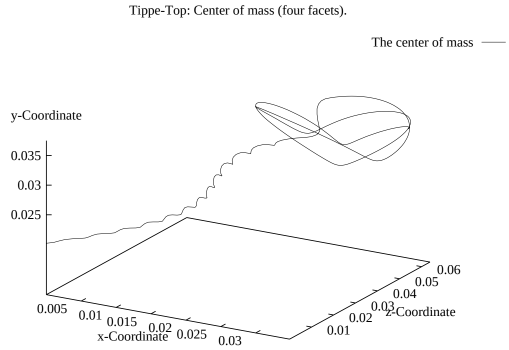


Figure 4: The three-dimensional lifting-motion of the center of mass shows how the tumbling motion of the top gets stronger with decreasing rotational energy.

After the inversion of the tippe-top at time $t \approx 0.6$ the energy only decreased very slowly in both simulation approaches. This problem was resolved by introducing an approximative modeling of rolling friction. In the case of rolling friction the contact point

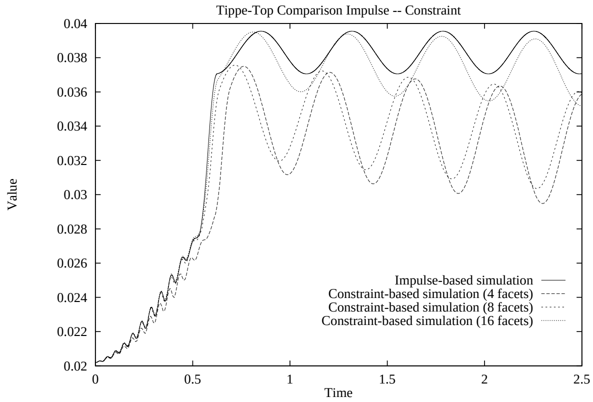
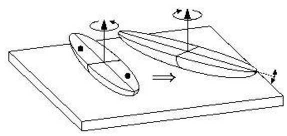
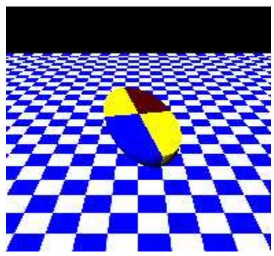


Figure 5: The normal coordinate of the center of mass as a means to compare both simulation types. The uppermost curve corresponds to the impulse-based method, the curves below to the constraint-based method with sixteen facets, then with eight and four facets.

velocity $\dot{\boldsymbol{p}}$ disappears: $\boldsymbol{v} = \boldsymbol{r} \times \boldsymbol{\omega}$. The Coulomb-law for dynamic friction determines the friction force as $\mathbf{F}_t = -\mu|\mathbf{F}_n|\frac{\dot{\boldsymbol{p}}_t}{|\dot{\boldsymbol{p}}_t|}$. So for $\dot{\boldsymbol{p}} \to 0$ also $\mathbf{F}_t \to 0$ holds. In this case the object looses very little energy. The effect is that after the overturn of the top, it keeps on moving in its inverted orientation for an unrealistically long time. The Kane and Levinson approach had the same problem. Therefore we introduced, according to Lewis and Murray [LM95], two more forces in the case of rolling-friction. They are a very approximative way to take account of this special situation: 1. A modified friction force $\mathbf{F}_t = -\mu_R m \boldsymbol{g} \frac{\boldsymbol{v}_t}{|\boldsymbol{v}_t|}$ with μ_R being a rolling friction coefficient and $\boldsymbol{v}_t$ being the velocity of the center of mass in the contact plane. 2. An easy model of air resistance resulting in the torque $\mathbf{M}_f = -\mu_A \boldsymbol{\omega}_t$ with μ_A being an air resistance coefficient. With the aid of these two additional forces the object behaviour of the sphere and the tippe-top is much more realistic than before. Because the new forces are always dissipative, the energy consistency of both approaches still holds.

4 THE SIMULATION OF THE RATTLEBACK

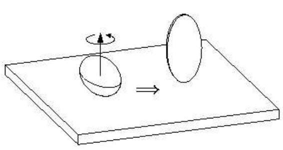


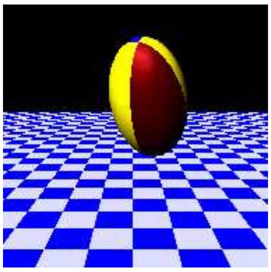


The name Celtic wobblestone probably stems from ancient times when Celtic priests used wobblestones as a kind of oracle. It is, in the physical sense, a top that is not rotational symmetric to its figure axis and therefore exhibits a very strange behaviour. The rattleback has a smoothly curved lower surface and when spun on a smooth horizontal surface it appears to be biased for one spin direction. If initially spun in the opposite direction, it will reverse its spin direction (see the left picture above). But there are two self-induced oscillatory motions as well that arise at the point of reversion. As you can see in figures 7 and 8 the maximum oscillations occur a short time before the spinning direction changes. At the time of reversion the oscillations already decay. There is not much literature on that topic, for an overview see Garcia and Hubbard [GH86]. We were interested in the rattleback for two reasons: first of all as another evaluation example and secondly because of its quadratically bounded surface that forced us to generalize our constraint-based algorithm. Dependent on the initial spinning velocity and the initial configurations multiple reversals can occur. Our two rigid body approaches also simulated this effect.

5 THE SIMULATION OF A SPINNING ELLIPSOID AND A SPINNING COIN WITH A HOLE

In this section we mention two additional simulations from the field of classical mechanics that we carried out.





The first one shows an ellipsoid spinning on a plane. The result can also be observed by spinning a boiled egg lying on a table as sketched above. The egg will stand up and spin along its largest symmetry axis. The reason for this behaviour is not an eccentricity of the center of mass, it is the geometry of the object and the moment arm induced by it. The spinning ellipsoid simulation is very sensitive to an increase in stepsize. We assume the curved boundary to be the reason for this effect. The approach of Kane and Levinson produced attracting forces in this example. Therefore a simulation of the motion with their technique was impossible.

We carried out another experiment that has the same physical background as the sphere with eccentric center of mass or the tippe-top. If you spin a coin that has a hole above its center in its initial upward position, the hole will travel downwards. See the textured and ray-traced pictures below to get an impression of what is happening.

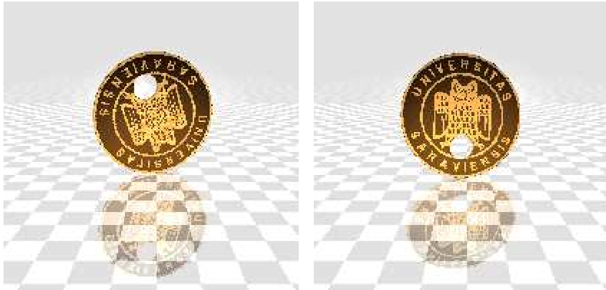


Figure 6: See also the homepage of the SiLVIA-system of the Universität des Saarlandes in Saarbrücken: `http://www-hotz.cs.uni-sb.de/silvia/silvia.html`.

6 CONCLUSION AND FURTHER WORK

Even if the above examples rather seem to be a matter of basic research, they very well served as a means to evaluate the reliability of both simulation techniques in regard to physical correctness as well as computing time. The physical reliability of the friction-modeling and the spent computing time are well-balanced in both approaches. Therefore both methods are predestined for the use in virtual reality applications (focus on real-time performance) as well as for the use in engineering applications (focus on 'physical correctness'). The only drawback of the approaches is that they are mathematically much more difficult and harder to implement and to test e.g. as the classical spring force approaches, where the depth of interpenetration of the objects determines the spring constants. But these methods are known to be physically incorrect and numerically problematic. Beyond that the approach to use constraint forces or impulses for unilateral contact conditions is to be preferred to reduced coordinate methods.

In our GALILEO-system we will in a next step combine both methods to a hybrid system. Furthermore the friction pyramid must be scaled according to the remaining computing time. The impulse-based method can be speeded up by using algebraic collision laws instead of complicated differential equations. These are of course physically less accurate, but have to be seen in the context of 'time-critical computing'.

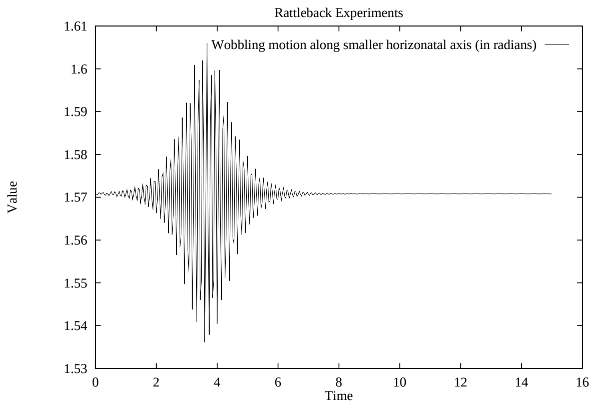


Figure 7: The oscillatory motion about the shorter horizontal symmetry axis measured in radians.

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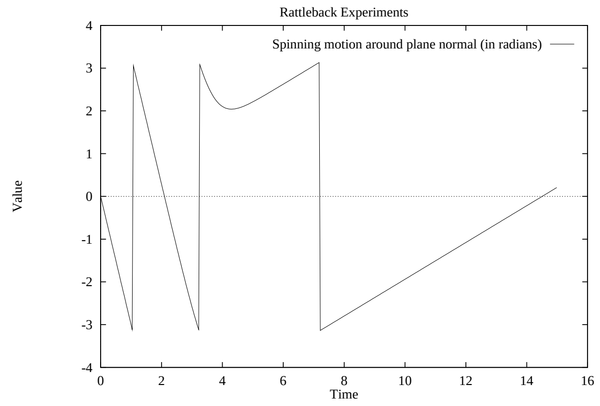

Figure 8: The reversion of the spin direction at time $t \approx 4.5$ described by the rotation of the rattleback about the plane-normal.

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